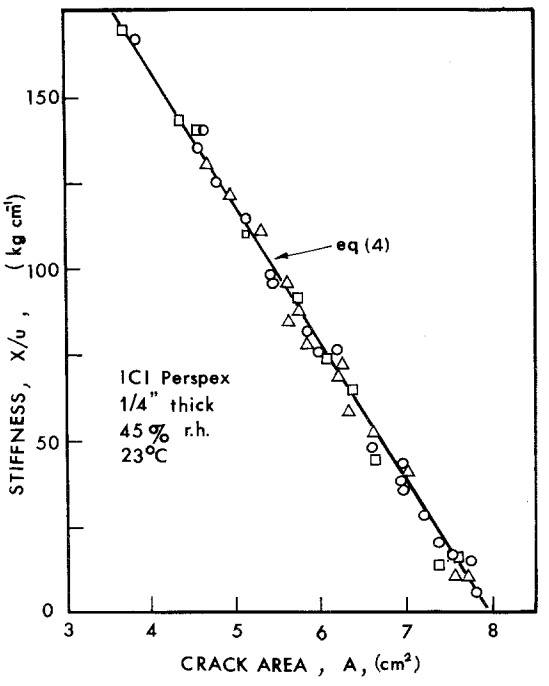

Figure 4 Variation of stiffness (X/u) with crack area (A) for the CDCB specimen with $m = 4$ in.$^{-1}$.

the adhesive joint, for low m specimen profiles and large crack lengths, by just using the load at fracture, this must necessarily give erroneous R-values. The correct R results can only be obtained using Equation 3. Of course, the irreversible work area method of Gurney [5, 7–10] is also valid for R measurements in this situation.

In view of the increasing use of these CDCB specimens with small m values (for improvement of beam stiffness and for avoidance of crack turning from the fracture plane), for fracture toughness determination of adhesive joints, we hope that this communication should be of some interest to experimentalists. A more detailed discussion on the stability of cracking in CDCB specimens has also been presented elsewhere [11, 12].

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Brittleness as an indentation size effect

It is well known that the mechanical response of certain solids can change dramatically with such variables as temperature, strain-rate, etc. The classic manifestation of this mechanical variability is the "ductile–brittle transition" evident in many engineering materials. Here we consider the influence of one largely unexplored variable, the scale of the overall deformation process, on the degree of brittleness. Standard indentation testing techniques provide a convenient basis for quantifying the effect.

The idea developed here derives from the observation that well defined hardness impressions may be produced in the hardest of solids at sufficiently low loads, but that the incidence of cracking about these impressions increases as the load level is raised. While considerable attention has been devoted to analysis of the residual

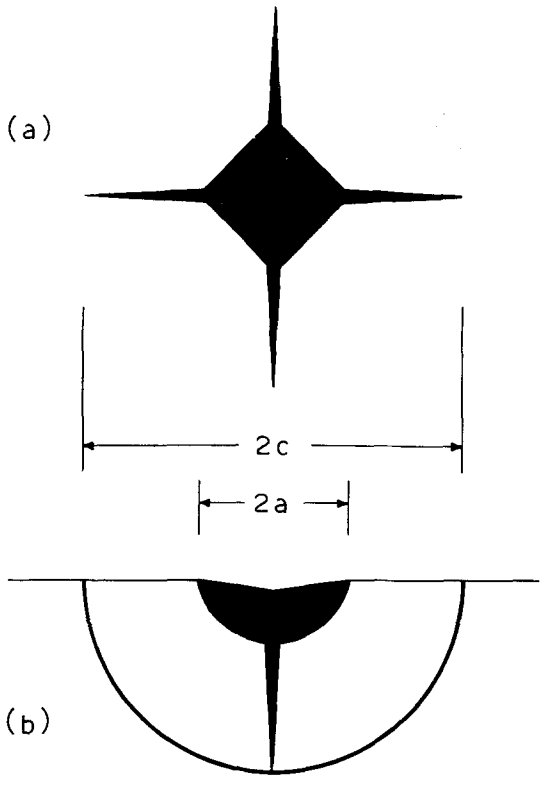

Figure 1 Indentation pattern, showing (a) plan and (b) side views, for Vickers diamond pyramid and homogeneous, isotropic specimen. Deformation indicated by central, dark region, fracture (so-called "median" cracks) by full, heavy lines. (At relatively high loads so-called "lateral" cracks, associated with residual stresses about the deformation zone, begin to cause some disruptive chipping [3].)

impressions themselves [1, 2], particularly in relation to such irreversible deformation modes as plastic flow and structural densification, it is only recently that systematic studies have been made of the attendant crack patterns [3]. Simple formulations are now available for the characteristic dimensions of both deformation and fracture zones as a function of contact load, in terms of basic material parameters, thereby opening the way to the evaluation of a transition size factor.

For simplicity, we consider indentation patterns which, in the extremes of either pure deformation [1] or pure fracture [4], are governed by conditions of geometrical similarity. Fig. 1 shows the pattern in the case of a standard Vickers diamond pyramid indenter, for an intermediate situation where both residual impression and crack traces are evident. In such a case the indenter load P relates to the characteristic dimension a (Fig. 1) of the residual impression according to

$$P/a^2 = \alpha\pi H \qquad \text{(deformation)} \qquad (1)$$

via the contact pressure H, which for a homogeneous material affords a convenient measure of the hardness; α is a dimensionless factor which depends on indenter shape (e.g. $\alpha = 2/\pi$ for Vickers pyramid). Empirically, it is found that this relation is not significantly affected by the onset of fracture, provided the impression remains well defined. On the other hand, once indentation-induced fracture reaches the well developed stage, such that the cracks extend well beyond the deformation zone on near-circular fronts (i.e. become "penny-like" [4]), the operative indentation relation becomes

$$P/c^{3/2} = (2\Gamma E/\kappa)^{1/2} \qquad \text{(fracture)}, \qquad (2)$$

with P now connected with the characteristic

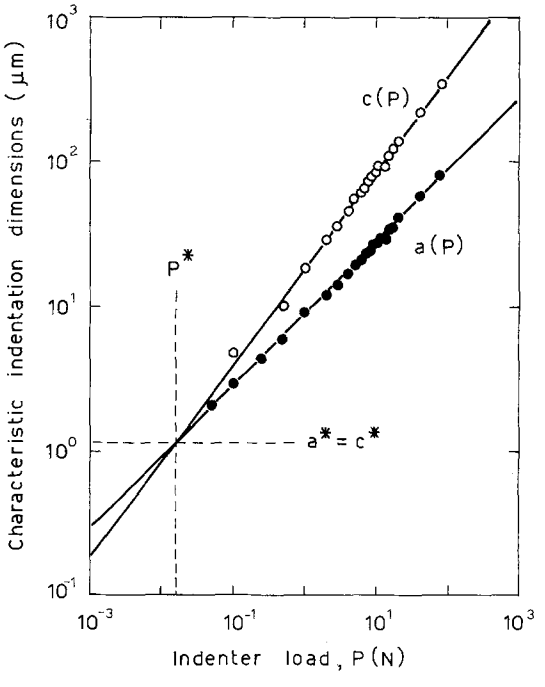

Figure 2 Indentation data for Vickers pyramid on soda-lime glass, at S.T.P., loading time 15 sec. Interval between indentation and subsequent measurement > 30 min (to allow system to come to equilibrium with environment). Lines fitted to $a(P)$ and $c(P)$ data on logarithmic plot with slopes $\frac{1}{2}$ and $\frac{2}{3}$ respectively.

dimension c of the crack, via the fracture surface energy Γ and Young's modulus E; κ is another dimensionless factor which depends primarily on indenter shape (notably on the characteristic included angle at the tip), but which also involves Poisson's ratio of the indented material in a minor way [4]. The validity of Equation 2 is not expected to extend down to the earlier, initiation stages of the fracture.

To illustrate the application of Equations 1 and 2, we plot in Fig. 2 the appropriate functions $a(P)$ and $c(P)$ for a commercial soda-lime glass. The intersection point of these two functions (obtained by extrapolation of working-range data in the particular example of Fig. 2) suitably characterizes the transition from deformation-dominated to fracture-dominated behaviour with increasing load. Designating this intersection point by asterisk notation, we may combine the two equations to obtain

$$a^* = \xi\Gamma E/H^2 = c^* \qquad (3)$$

where $\xi = 2/\pi^2\alpha^2\kappa$ is a dimensionless, largely geometrical factor. The quantity $H^2/\Gamma E$ may accordingly be seen as an index of brittleness for any given indentation configuration, in the sense that cracking will tend to increase, at the expense of deformation, with the value of this quantity. Solids of high hardness, low fracture energy (notably covalent-ionic solids) will thus tend to exhibit contact fracture more readily than their opposites (e.g. metallic, polymeric solids).

The above procedure presents itself as a particularly simple contivance for investigating the parameters which control brittleness. In the interest of accuracy it would seem advisable to evaluate the key dimension a^* as in Fig. 2 from measurements over a wide range of applied loads, although in principle an estimate may be obtained from a single indentation by using an alternative combination of Equations 1 and 2,

$$a^* = a(a/c)^3. \qquad (4)$$

It is then a straightforward matter to follow changes in a^* with any given experimental variable. (Thus, for instance, one could readily evaluate the effect of composition on the brittleness of silicate glasses.) In such comparative studies it would be important to maintain constant test conditions: the sensitivity of ξ to indenter geometry, of H, Γ and E to specimen inhomogeneity or anisotropy, and of H and Γ to kinetic effects, might well lead to significant errors in an insufficiently controlled experiment. More absolute information on the basic material parameters involved here, particularly hardness and fracture energy, rests with our ability to determine (either theoretically or by experimental calibration) the dimensionless constants in Equations 1 and 2 [4].

From a more practical standpoint, the existence of a size effect in the indentation response has implications in a wide variety of technological problems. One important example concerns the wear of ceramics components [5]: the very mechanism of wear must be expected to depend on the scale of individual micro-indentation, surface-removal events. Accordingly, for typical glasses we should predict from Fig. 2 that fine-scale contact ($a \gtrsim 1\,\mu$m) by chipping (abrasion removal by ploughing (polishing mode), coarse-scale contact ($a \lesssim 1\,\mu$m) by chipping (abrasion mode). This prediction is in accord with available experimental evidence [6]. Consideration of such scale effects could prove especially relevant in the evaluation of efficiency and quality of surface finishing processes.

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